MATHEMATICS MODEL EXAM-ANSWER KEY

1. (a) $AB=\begin{bmatrix} 8 & 6 & 3 \\ 20 & 12 & 4 \end{bmatrix}$ [1 Mark]

(b) BA is not defined. No.of columns of B ¿ No.of rows of B [1+1 marks]

2. $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$ $C_1 \to C_1+C_2+C_3$

[1]

$$(5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix} = (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & 4-x & 0 \\ 0 & 0 & 4-x \end{vmatrix} = (5x+4)(4-x)^2$$

[1+1]

3. $\int \frac{(x-3)}{(x-1)^3} e^x dx = \int \left(\frac{x-1}{(x-1)^3} - \frac{2}{(x-1)^3} \right) e^x dx = \int \left(\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right) e^x dx = \frac{e^x}{(x-1)^2}$ [3]

4. $\vec{a} = 4\hat{i} + 8\hat{j} + \hat{k} \hat{\Rightarrow} \hat{a} = \frac{4}{9}\hat{i} + \frac{8}{9}\hat{j} + \frac{1}{9}\hat{k} \Rightarrow D.COSINES$ ARE $\left(\frac{4}{9}, \frac{8}{9}, \frac{1}{9}\right)$

[1]

$\alpha = \cos^{-1}\left(\frac{4}{9}\right)$, $\beta = \cos^{-1}\left(\frac{8}{9}\right)$, $\gamma = \cos^{-1}\left(\frac{1}{9}\right)$ [2]

5. (a) Order =2 degree = 1 [1]

(b) $\frac{dy}{dx} = -a\sin x + b\cos x, \frac{d^2y}{dx^2} = -a\cos x - b\sin x \Rightarrow \frac{d^2y}{dx^2} = -y \; i\,e \; \frac{d^2y}{dx^2} + y = 0$ [2]

6. $(a)\sum_{i=1}^{n} p(x_i)=1 \quad \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+p=1 \Rightarrow p=\frac{1}{16}$ [1]

(b) $Mean\, E(x)=\sum_{i=1}^{n} x_i p_i \Rightarrow 1\times\frac{1}{2}+2\times\frac{1}{4}+3\times\frac{1}{8}+4\times\frac{1}{16}+5\times\frac{1}{16}=\frac{31}{16}$ [2]

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7. $\vec{r}=(2i+j-3k)+\lambda(2i+5j-3k) \wedge \vec{r}=(-2i+4j+5k)+\mu(-i+8j+4k)$ [1]

(b) $\cos\Theta=\dfrac{\vec{b_1}.\vec{b_2}}{|b_1||b_2|=\frac{(-2)+40-12}{\sqrt{38}.\sqrt{81}}=\frac{26}{9\sqrt{38}}¿}$ ¿ [2]

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8. $R=\{(1,1),(2,2),(3,3),(1,3)(3,1)(2,3)(3,2)\}\, on\, A=\{1,2,3\}$

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9. $(a)\, LHL=\log_{x\to ¿2} f(x)=3, \log_{x\to 2^{+¿} f(x)=3, f(x)\, is\, continuous\, at\, x=2¿}$

Hence $f(x)\, is\, continuos\, every\, where.$

$LHD\, at\, x=2\, is\, 1 \wedge RHD\, at\, x=2\, is -1 \therefore LHD \neq RHD$

$f(x)\, is\, not\, differntiable\, at\, x=2$

10. (a) $y=\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\frac{5\pi}{6}$ [1]

(b) $\tan^{-1}\frac{1}{5}+¿\tan^{-1}\frac{1}{7}+¿\tan^{-1}\frac{1}{8}+¿\tan^{-1}\frac{1}{3}=\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{35}}\right)+\tan^{-1}\left(\frac{\frac{1}{8}+\frac{1}{3}}{1-\frac{1}{24}}\right)¿¿¿$

$\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{35}}\right)+\tan^{-1}\left(\frac{\frac{1}{8}+\frac{1}{3}}{1-\frac{1}{24}}\right)=\tan^{-1}\left(\frac{12}{34}\right)+\tan^{-1}\left(\frac{11}{23}\right)=\tan^{-1}1=\frac{\pi}{4}$

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11. $put\ x=tan\theta\ y=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{tan\theta}\right)=\tan^{-1}\left(\frac{sec\theta-1}{tan\theta}\right)=\tan^{-1}\left(\frac{1-cos\theta}{sin\theta}\right)$

$\tan^{-1}\left(\frac{1-cos\theta}{sin\theta}\right)=¿\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)=\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}=\frac{\tan^{-1}x}{2}¿$

(b) $\frac{dy}{dx}=\frac{1}{2\left(1+x^2\right)}$

12. $(a)\ 2:5:-1$

$\textit{mid point of } AB \textit{ is} \left(2,\frac{3}{2},\frac{3}{2}\right)$

$\textit{equation of the plane is } a\left(x-x_1\right)+b\left(y-y_1\right)+c\left(z-z_1\right)=0$

$2x+5y-z=10$

13. $\int_a^b f(x)=\lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots\ldots..f(a+(n-1)h)\right]$

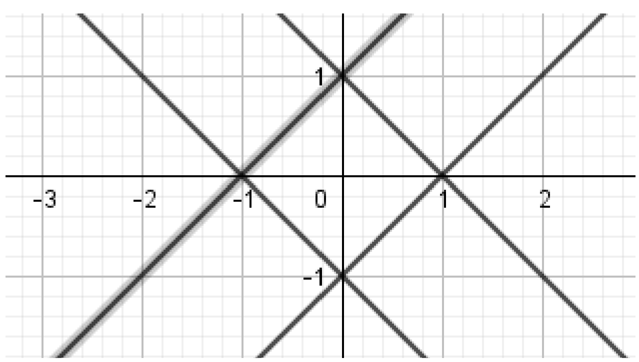
$$\int_{-1}^{1} e^x\,dx=e-e^{-1}$$

14. (a) 0 (b) $\int_0^4 |x-1|\,dx=5$ [4]

15. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

(a) $p=\frac{1}{10}$ (b) $p=\frac{1}{5}$ [2+2]

16. (a)



[2]

Total area $=4\times\int_0^1 (1-x)\,dx=2\,sq\ unit$ [2]

17. (a) $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$ [1]

(b) $(AB+BA)^T=(AB)^T+(BA)^T=B^T A^T+A^T B^T=BA+AB=AB+BA$

$AB+BA$ *IS SYMMETRIC* [3]

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18. (a) $\vec{a}\times\vec{b}=7i-2j-3k$ [2]

(b) $Area=\frac{1}{2}(\vec{a}\times\vec{b})=\frac{1}{2}\sqrt{62}$

© $Required\ vector=\frac{7i-2j-3k}{\sqrt{62}}$ [1]

19.(a) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ [1]

(b) $|A|=1(1+3)+1(2+3)+1(2-1)=4+5+1=10\neq 0 \therefore consistent$ [2]

© $adjA=\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} X=A^{-1}B\wedge A^{-1}=\frac{adjA}{|A|}$ [2]

$X=\frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}\Rightarrow\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\Rightarrow x=2, y=-1\Rightarrow, z=1$ [1/2+1/2]

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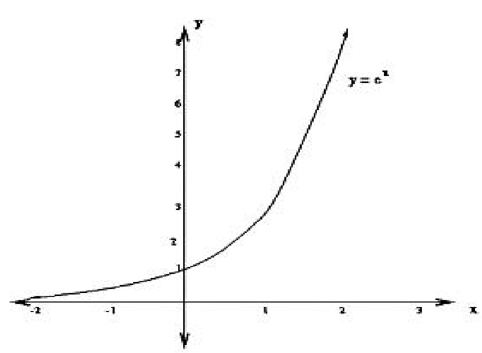
20. (a) graph of e^x [1 mark]



(b) $f(x)=e^x\Rightarrow f'(x)=e^x>0$ for every x

$f(x)$ *is strictly increasing function* [2 marks]

© $y=x^{\frac{1}{x}}$

$$logy=\frac{logx}{x}\Rightarrow\frac{dy}{dx}=\frac{x^x(1-logx)}{x^2}\therefore\frac{dy}{dx}=0\Rightarrow1-logx=0\Rightarrow logx=1\Rightarrow x=e$$

$\frac{d^2y}{dx^2}<0\, when\, x=e\, i\, e\, y\, is maximum\, when\, x=e$ [3 Marks]

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21. (a) $\frac{dy}{dx}=\frac{-(2x+2y)}{(2x+2y)}=-1$[2 Marks]

(b) $\frac{dy}{dx}=\frac{-[y^x logy-2^x\log2]}{x y^{x-1}}$ [2 Marks]

© $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-cos\theta}{sin\theta}=-cot\theta=-\cot\frac{\pi}{4}=-1$ [2 Marks]

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22. (a) $\frac{2x}{9}+\frac{2y}{16}\frac{dy}{dx}=0\, since\, tangent\, is \| ¿\, x-axis\,\frac{dy}{dx}=0$ [1]

$\therefore\frac{2x}{9}=0\, since\,\frac{dy}{dx}=0$ [1]

$\therefore x=0\, Then\, given\, equation\, becomes\,\frac{y^2}{16}=1\Rightarrow y^2=16\Rightarrow y=\pm4$ [1]

$The\, required\, points\, are\,(0,4)\wedge(0,-4)$

(b) $area\, of the\, ellipse\; A=4\times\int_0^3 ydx=4\times\frac{4}{3}\int_0^0\sqrt{9-x^2}$ [1+1/2]

$A=\frac{16}{3}\left[\frac{x}{3}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3=\frac{16}{3}\times\frac{9}{2}\times\frac{\pi}{2}=12\pi$ [1+1/2]

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23. $a*b=a+b-ab\wedge b*a=b+a-ba=a+b-ab=a*b$ [2 marks]

¿ *is commuative*

(b) Let e be the identiy elemnt $\therefore a*e=a \Rightarrow a+e-ae=a \Rightarrow e=0$ [2 Marks]

(c) Let the inverse of 3 be x. then $3*x=e \Rightarrow 3+x-3x=0 \Rightarrow 3-2x=0 \Rightarrow x=\frac{3}{2}$ [2]

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24. Given equation can be written as $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{cotx}{1+x^2}$

(a) Integrating factor $I.F=e^{\int \frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$ [2 Marks]

(b) Solution is given by $y(1+x^2)=\int \frac{cotx}{1+x^2}(1+x^2)\,dx$

$y(1+x^2)=\log|sinx|+C$ [2 Marks]

(c) Given $y=0$ when $x=\frac{\pi}{2}$

$y(1+x^2)=\log|sinx|+C \Rightarrow 0\times(1+x^2)=\log\left|\sin\frac{\pi}{2}\right|+C$

ie $0=0+C$ i e $C=0$ particular solution is $y(1+x^2)=log|sinx|$ [2 marks

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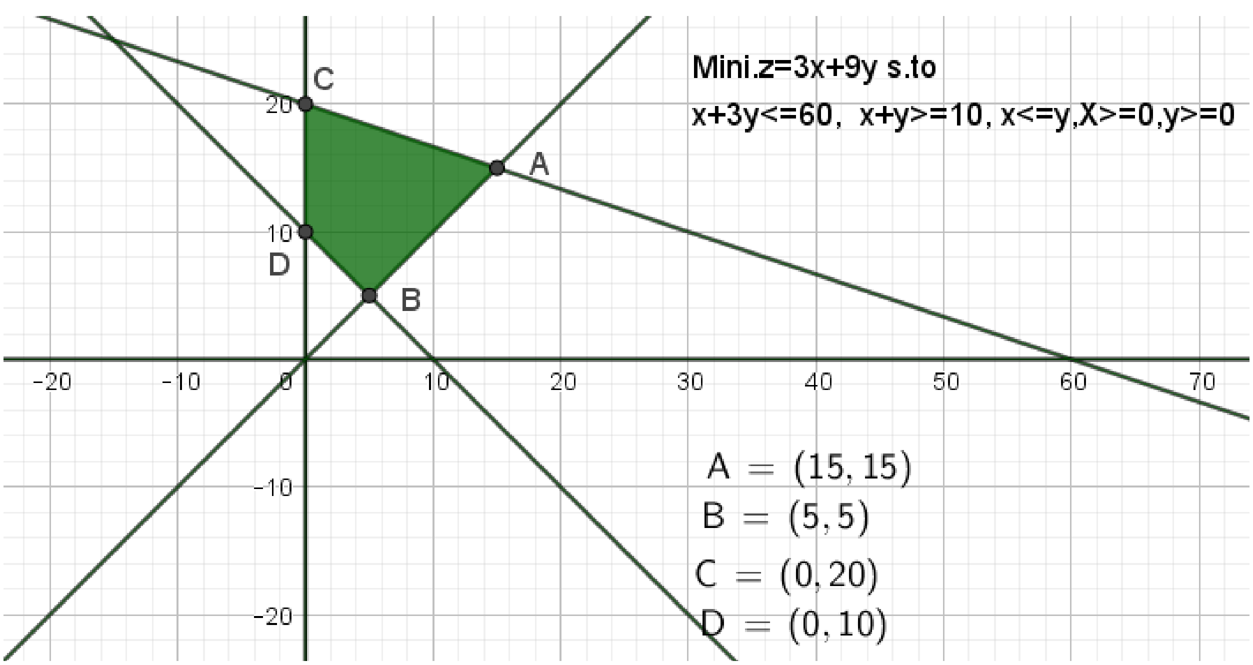

25.

Mini. value of Z at C (0.20) = $3 \times 5 + 9 \times 5 = 60$

(1) Value of $Z\,at\,A(15,15)\,is\,Z = 3 \times 15 + 9 \times 15 = 12 \times 15 = 180$

(2) $Value\,of\,Z\,at\,B(5,5)\,is\,Z = 3 \times 5 + 9 \times 5 = 60$

(3) $Value\,of\,Z\,at\,C(0,20)\,is\,Z = 3 \times 0 + 9 \times 20 = 180$

(4) $Value\,of\,Z at\,D(0,10)\,is\,Z = 3 \times 0 + 9 \times 10 = 90$